3701. Use small-angle approximations to write $\mathrm{f}(x)$ as a quartic polynomial. Then differentiate twice.

3702. (a) Pick e.g. $\mathbf{a} = \mathbf{i} + \mathbf{j}$. For the spiral, visualise the graph as $(\cos p, \sin p)$, which is a unit circle, scaled by an increasing enlargement $\times p$.
   (b) Let $\mathbf{a} = c\mathbf{i} + d\mathbf{j}$. Set up simultaneous equations. Solve these using the method for equations of the type $R \sin \alpha = k_1$, $R \cos \alpha = k_2$ (as when writing in harmonic form).

3703. Integrate by substitution, with $u = 1 + \sqrt{x}$.

3704. Set up the cosine rule for c^2. Differentiate wrt t.

3705. No calculations are needed here. Consider the fact that all values of x^2 are positive.

3706. Rearrange to make x the subject.

3707. (a) The relevant fact is that the string is smooth.
   (b) Find the interior angles of the string triangle, using the cosine rule. Then, considering the angle bisectors, find the angle of inclination of the forces of magnitude F N. Then resolve horizontally and solve for F.

3708. Take a factor of i out of the inner sum. You can do this because, as far as the inner sum is concerned, i is a constant. Then you can evaluate the inner j sum as an arithmetic series.

3709. Find the equation of the tangent at $\left(m, m^2\right)$. Then substitute in (a, b) and solve a quadratic in m.

3710. Use the conditional probability formula. Find the probabilities using a calculator's normal facility.

3711. (a) When you apply g repeatedly to x, you get inlaid fractions. Deal with these in the usual way, multiplying top and bottom of the main fraction by the denominator(s) of the inlaid fraction(s).
   (b) For g^n to be well defined, you require all of $\mathrm{g}, \mathrm{g}^2, ..., \mathrm{g}^n$ to be well defined. Division by zero excludes various numbers from the domain.

3712. Substitute into $x^2 + y^2$, and simplify to get 1.

3713. Let $z = \tan x$ and $y = \cot x$. Then find $\frac{dz}{dy}$ using the chain rule.

3714. Either use the discriminant $\left(\text{of quadratics in } x^2\right)$, or else sketch the curves.

3715. Find the time and speed at which the first ball hits the ground. Set this to be $t = 0$, and then equate two expressions for height, each in terms of t.

3716. Set up an identity, such that $\left(x^3 + 3x^2\right) \div \left(x + 2\right)$ is equivalent to $ax^2 + bx + c + \frac{d}{x+2}$. Then multiply by $x + 2$ and equate coefficients.

3717. Let $y = \mathrm{f}(x)$. Then separate the variables x and y and integrate. At the end, when you get e^{kx}, you can simplify further.

3718. Factorise the numerator, noting (either by using the factor theorem or by the "meta-fact" that the question exists) that $p - q$ must be a factor.

3719. In both parts, drawing a number line (of outputs) may help. Consider the greatest and least possible values of the outputs.

3720. (a) Use log rules to simplify.
   (b) Use calculus: set the first derivative to -1. You can't easily use the discriminant method, because the equation for intersections is cubic, and its roots are unknown.

3721. Place the first tile wlog. Consider the diagonally opposite tile. Then consider the other two tiles, one by one.

3722. Consider the shape of the curve as $x \to k^-$, i.e. x tends to k from below.

3723. The parabolae are reflections in $y = x$. So, look for intersections of the first parabola and that line.

3724. Express the numerator as $x^2 + 8x - 9 + 18$, and split the fraction up. Then complete the square on the denominator. Find the range of the denominator, then the fraction, then f.

3725. Find the equation of a generic tangent at $x = a$. Substitute $x = a - 6$ and $y = (a - 6)^3 - (a - 6)$ into this and solve for a.

3726. Subtract the equations and solve for x. This will show you the plane in which the intersections lie. Substitute back into the equation of S_1 to get a (y, z) circle.

3727. (a) Consider only the lower bank. The horizontal forces cancel out for all but the right-hand peg. Resolve the two tensions horizontally.
   (b) Find, in terms of n, the number of sections exerting a vertical force on the lower bank.

3728. Consider input transformation: replace y by ky.

3729. Simplify with an identity and then use the small-angle approximation for tan.

—— Alternative Method ——

Find the equation of the tangent to $y = \cot\left(\theta - \frac{\pi}{2}\right)$ at $\theta = 0$.

3730. Call the *squared* lengths a, b, c. Then set up three simultaneous equations and solve.

3731. (a) Set e.g. $a = 2$ and sketch that specific case.
(b) Find the equation of a generic tangent at A, and show that $|OY| = |AP|$.

3732. Let $y = \ln x$, so that $x = e^y$. Then differentiate implicitly with respect to x.

3733. (a) Find the first and second derivatives, and sub them into the LHS of the DE.
(b) Set the first derivative to zero. Solve this with the N-R method, choosing $t_0 = 0$.
(c) Consider the limit of the exponential term as $t \to \infty$.

3734. In each case, take out a factor of $a - x$ from the denominator.

3735. (a) You can write down the answer.
(b) And again.
(c) Either use the reverse chain rule, or (same thing explicitly) the substitution $u = 3x$.

3736. Square both sides and simplify.

3737. Use the substitution $t = 2\theta$. Then use a $\cos 2\theta$ double-angle formula to simplify the square root and thus integrate.

3738. For the tangents to cross the curve and intersect it only once, the points of tangency must be points of inflection. Find the first and second derivatives, using the quotient rule. Set the second derivative to zero, and find the points of inflection. Then find the equations of the tangent(s). By symmetry, you only need find one of them.

3739. (a) Consider ae^{kt} as a (variable) scale factor of enlargement. Since $x = \cos t$, $y = \sin t$ is a point on the unit circle, rotating anticlockwise around the origin, the spiral is generated by scaling the radius.
(b) Using the parametric differentiation formula, set $\frac{dy}{dx} = 1$ and rearrange for $\tan t$.

3740. Calculate $\sum x$ and $\sum x^2$ for the sample of 100 and the set of 20, and so the remaining sample of 80.

3741. Use $F = ma$. Then consider a in harmonic form. (You don't need the angle, only the amplitude.)

3742. You only need the first two terms of each of the expansions. Write these down, then multiply out and form simultaneous equations in m and n.

3743. (a) The line has an endpoint because $\sqrt{x+y}$ is only defined for $x + y \geq 0$. So, set $x + y = 0$.
(b) Differentiate implicitly with respect to x, then rearrange to make $\frac{dy}{dx}$ the subject.
(c) Substitute $x + y = 0$ into $\frac{dy}{dx}$.

3744. (a) Find the positions of the two ships as vectors in terms of t, and use Pythagoras.
(b) Find the derivative of d^2 with respect to t, and set it to zero.

3745. Using a double-angle formula, find the common ratio. Use this to produce an expression for u_3 in terms of x. Then solve the resulting equation.

3746. The second equation is a circle. Find its centre and radius. You can then write down the answer without any calculation.

3747. Consider boundary cases, which can be thought of as maximal and minimal overlap between the events A and B. A Venn diagram may help.

3748. Multiply by $x^{\frac{1}{3}}$ to form a quadratic in $x^{\frac{1}{3}}$.

3749. Replace the variables x and y by...

3750. Take out a factor of $\sqrt{x}$ from the first bracket, and simplify using a difference of two squares. Set the first derivative to zero for SPs. Also, consider the behaviour at $x = 0$ and as $x \to \infty$.

3751. Consider the fact that four edges meet at every vertex.

3752. Consider three cases for the discriminant: $\Delta < 0$, $\Delta = 0$ or $\Delta > 0$. Show that two of these are possible. Apply this fact to all three equations.

3753. Use the fact that the angle between $y = mx$ and the x axis is $\arctan m$.

3754. Set up the equation of the circle. Then solve for intersections, substituting for x^2. Show that the origin is always a point of intersection, and then look for the values of r which guarantee that there will be no other intersections.

3755. Solve for intersections, using a polynomial solver. Then set up a single integral with respect to y for the area between the curves. Evaluate this with a calculator.

3756. Use L'Hôpital's rule to set up the new limit. Then use small-angle approximations.

3757. Differentiate implicitly with respect to x.

3758. (a) Write the sum longhand before multiplying out the brackets.
   (b) Little calculation is needed. The identity in part (a) contains the expression you need to evaluate, and you are given everything else!

3759. "Explain", in this case as often in mathematics, is easiest done algebraically. Try to find A and B, and find a contradiction.

3760. Circumscribe a circle. Then use circle theorems.

3761. Find and factorise the second derivative.

3762. (a) Use the normal facility on a calculator.
   (b) Give the distribution of $X_1 + X_2 + X_3$ first, then do as in (a).

3763. The parabola which best approximates $y = \cos x$ at $x = 0$ is $y = 1 - \frac{1}{2}x^2$. That point is a maximum. At $x = \pi$, the shape is the same, at a minimum. So, transform the parabola $y = 1 - \frac{1}{2}x^2$ such that its vertex moves from being a maximum at $(0, 1)$ to a minimum at $(\pi, -1)$.

3764. Let $\theta = \arctan x$ and $\phi = \arctan y$. Then start with the compound-angle formula for $\tan(\theta + \phi)$.

3765. Restrict the possibility space using the condition. Count the number of outcomes in which the red counters are together. Then, out of these, count the number of outcomes in which the blue counters are also together. Divide these.

3766. Draw a force diagram for the case with nine stones. There are three forces on the keystone. Work out the angle between each side of the keystone and the vertical, in terms of k. Resolve vertically.

3767. To get an accurate sketch, consider the following: if $f(x)$ has a single root at $x = \alpha$, then $y^2 = f(x)$ has a tangent parallel to the y axis at $x = \alpha$.

3768. The first statement is false; the second is true.

3769. Since the AP is symmetrical about b, the graphs are symmetrical in the line $x = b$. Use this fact to find the multiplicity of the root at $x = b$.

3770. Consider the region in the middle as an equilateral triangle plus three segments. Find the area of the segments in the usual manner, as a sector minus a triangle (the same equilateral triangle).

3771. (a) Consider that $x = p$ is a root.
   (b) Cancel the relevant factor, then take the limit.

3772. Draw (in 2D) the triangle with vertices at $(0, 0, 0)$, $(1, 1, 0)$ and $(1, 1, 1)$. Find its side lengths.

3773. Sketch the curves first, in order to establish what needs to be shown.

3774. Find first and second derivatives by the product rule, and substitute into the LHS. Simplify to reach the RHS.

3775. Since $(2, 5)$ is a stationary point of inflection, the cubic must take the form $y = k(x - 2)^3 + 5$.

3776. In each case, just solve the first equation, and see whether the roots satisfy the second equation.

3777. For fixed points (period 1), $f(x) = x$. So, the equation for points fixed by $f^2(x)$ (period 2) is $f^2(x) = x$. However, solving for period 2 will also find any solutions to period 1.

3778. The question boils down to "Sketch the parametric equations $x = a$, $y = 1/a$."

3779. Write $\sin x$ as $\cos x \tan x$.

3780. (a) Find $\mathbb{P}$(all three odd).
   (b) Consider the three possible cases, defined by the number of dice removed after the first roll.

3781. Factorise the first equation, taking out x^2 to leave a quadratic in x^2 and y^4.

3782. (a) Consider whether the numerator could also be zero at $x = \beta$.
   (b) Parity is the relevant fact.
   (c) Use (a) and (b), and also consider behaviour as $x \to \pm\infty$.

3783. Place a counter WLOG. There are nine successful locations for the second counter, and so on.

— Alternative Method —

Consider a possibility space of $^{16}C_4$ equally likely unordered outcomes.

3784. Since cosine is an even function, you can get rid of the modulus sign. Then, use the first Pythagorean trig identity to form a quadratic in $\cos x$.

3785. Looking for (the non-existence of) intersections, find an inequality linking a and b. Then consider this inequality as defining a region on (a, b) axes.

3786. Prove the reciprocal statement, using a compound-angle formula.

3787. Split the integrand into partial fractions, integrate, then solve the resulting equation.

3788. Consider only $x, y \geq 0$. Write y in terms of x, and substitute in. Optimise the resulting expression by setting the derivative wrt to x equal to zero.

3789. Write x and y in terms of t. Then substitute for t. You'll have $\sec^2 \theta$ in your answer. Use the second Pythagorean identity to convert this to $\tan^2 \theta$.

3790. Set the second derivative to zero, factorise and solve the resulting quadratic.

3791. Start by showing that h is a positive polynomial of even degree.

3792. This is a geometric series. By writing longhand, work out the first term and common ratio.

3793. This is false. It would be true if both f and g were polynomials. Look for a counterexample that isn't so well behaved.

3794. Let (a, b) be the vertex in the first quadrant. Show that the area of the rectangle can be expressed as

$$A = 8a\sqrt{1 - a^2}.$$

Set $\frac{dA}{da}$ to zero and optimise.

3795. Write the sum out longhand, substituting in the relevant values for r_1, r_2 and r_3.

3796. (a) Consider the values for which $\text{f}''(x) = 0$.
(b) Integrate once, and use the fact $\text{f}'(0) = 0$ to find the constant of integration. Integrate again, and use the fact $\text{f}(0) = 0$ to find the new constant of integration. At the end, use the fact that $\text{f}(x)$ is monic to find k.

3797. The best way of doing this is to imagine what would happen if a girder disappeared suddenly. Would the remaining structure fall in such a way as to increase the size of the gap left, or to decrease it? If the former, then the girder must have been in tension; if the latter, compression.

3798. (a) Write out the binomial probability tables for processes P_1, P_2. A two-way table constructed of these forms the possibility space.
(b) Do likewise for P_2, P_3.

3799. Only one of these is true. Consider $\text{f}(x) = x^2$.

3800. Find the equation of the tangent using the product rule. Then, to solve for re-intersections, write the expression $\sin x + \cos x$ in harmonic form.

—— End of 38th Hundred ——